10. Wave Optics

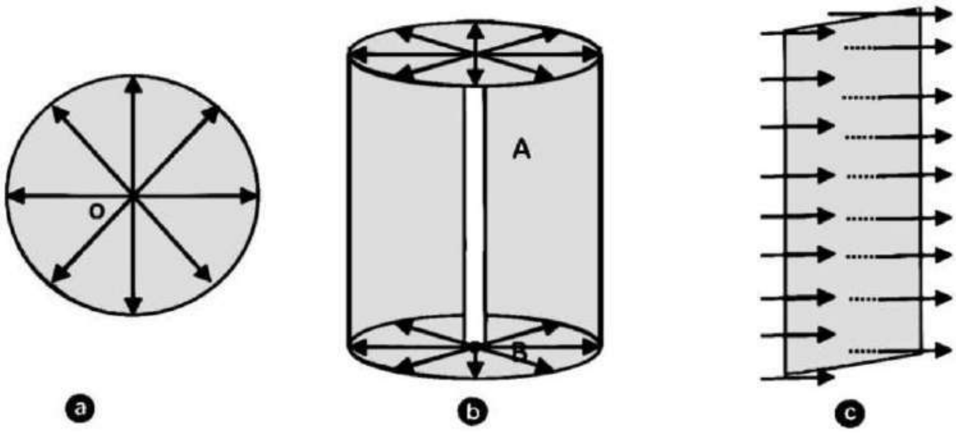
Wavefront - A wavefront is defined as the continuous locus of those particles of a medium, which are vibrating in the same phase.

(i) Spherical Wavefront -

When the source of light is a point source the wavefront is a sphere with centre at the source

(ii) Cylindrical Wave front -

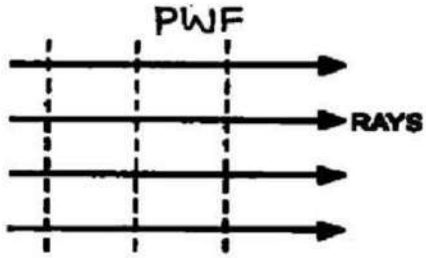
When the source of light is linear (a slit). then all the points equidistant from the source lie on a cylinder. Therefore the wavefront is cylindrical.



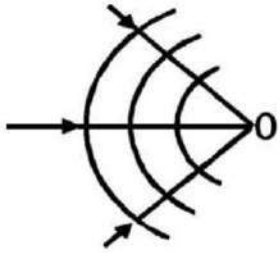
(iii) Plane Wavefront -

When the point source or linear source of light is at very large distance, a small portion of spherical or cylinderical wave front appears to be plan such a wavefront is called a plane wavefront.

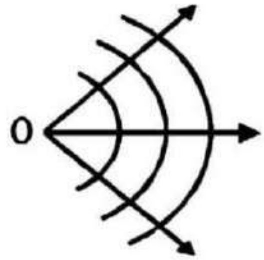
> * If we draw a set of straight lines which are perpendicular to the wavefront then these lines are called the ray of light.



plane wavefront



Converging spherical wavefront



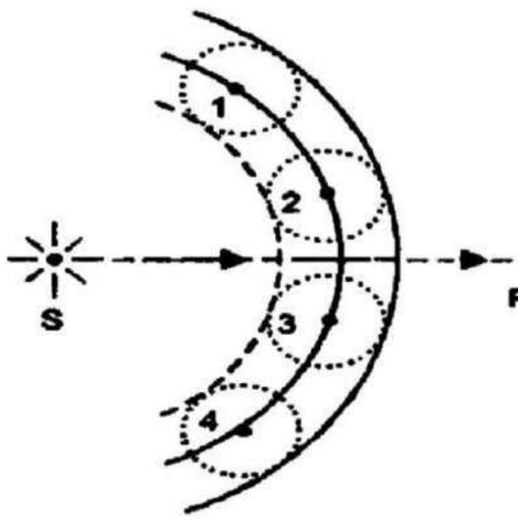
Diverging spherical wavefront

Huygen's Principle

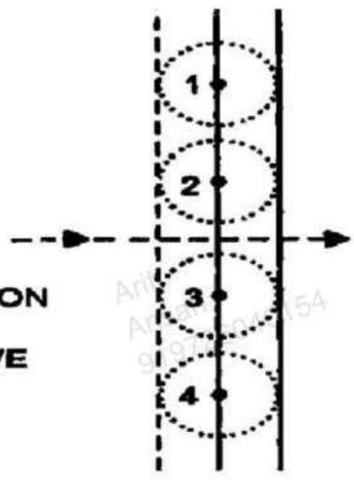
The Huygen's principle tells the way in which the wavefront is propagated further in a medium.

(i) Every point on the given wavefront (Called primary wavefront) acts as a fresh source of new disturbance, called secondary wavelets, which travel in all directions with the velocity of light in the medium.

(ii) A surface touching these secoundary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant. This is called secondary wave front.

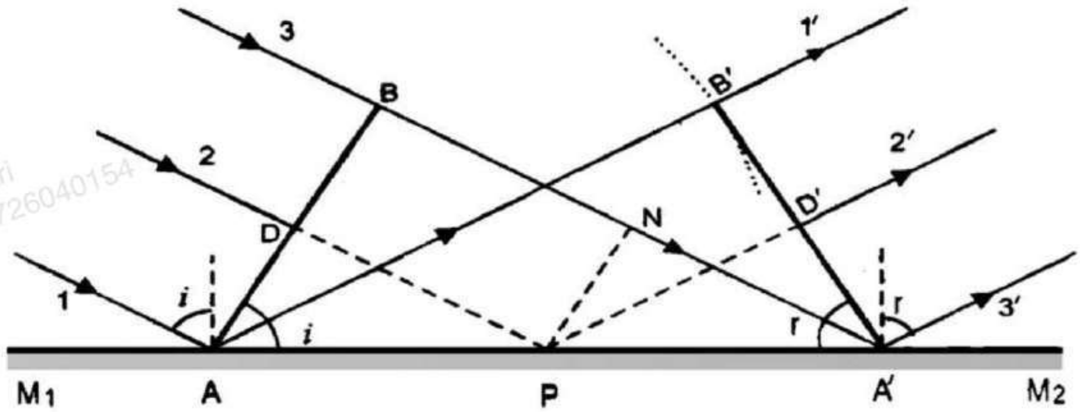


PROPAGATION OF LIGHT WAVE



Reflection on the basis of Wave Theory -

In the given figure AB is a plane wave front incident on a plane mirror M_1M_2 at $\angle BAA' = \angle i$. 1, 2, 3 are the corresponding incident rays, which are perpendicular to AB.



According to Huygens principle, every point on AB is a source of secondary wavelets. Let the secondary wavelets from B strike M_1M_2 at A' in t seconds.

$$\therefore \quad BA' = c \times t \qquad \text{——} \quad (1)$$

The secondary wavelets from A will travel the same distance $c \times t$ in the same time. Therefore, with A as centre and $c \times t$ as radius, draw an arc, so that -

$$AB' = c \times t \qquad \text{——} \quad (2)$$

From A' draw a tangent plane A'B' touching the spherical arc tangentially at B'. Therefore A'B' is the secondary wave front after t seconds. This would advance in the direction of rays 1', 2' and 3', which are the corresponding reflected rays perpendicular to A'B'.

Hence angle of incidence $i = \angle BAA'$

and angle of reflection $r = \angle B'A'A$

In $\Delta AA'B$ and $\Delta AA'B'$,

AA' is common, $BA' = AB' = c \times t$

and $\angle B = \angle B' = 90^\circ$

$\therefore$ $\Delta AA'B$ an $\Delta AA'B'$ are congruent.

$\therefore$ $\angle BAA' = \angle B'A'A$ i.e. $\angle i = \angle r$

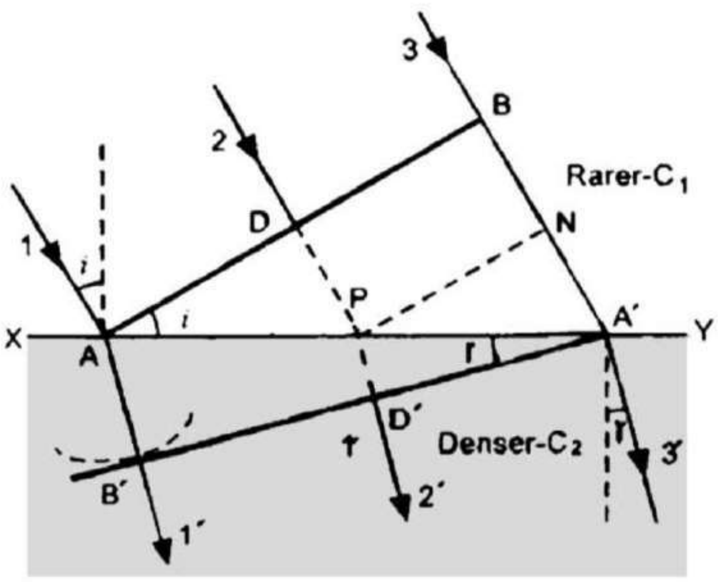
which is the first law of reflection.

Further the incident wavefront AB, the reflecting surface M_1M_2 and the reflected wavefront $A'B'$ are all perpendicular to the plane of the paper.

Therefore incident ray, normal to the mirror M_1M_2 and reflected ray all lie in the plane of the paper. This is second law of reflection.

Refraction on the basis of Wave Theory -

If v_1 is velocity of light in rarer medium and v_2 is velocity of light in denser medium then -



as $\mu_1 = \frac{c}{v_1}$ and $\mu_2 = \frac{c}{v_2}$

or $\boxed{{}^1\mu_2 = \frac{v_1}{v_2}}$ — (1)

Here AB is a plane wavefront incident on XY at $\angle BAA' = \angle i$. 1, 2, and 3 are the corresponding incident rays normal to AB.

According to Huygen's principle, every point on AB is a source of secondary wavelets.

Let the secondary wavelets from B strike XY at A' in t seconds.

$$\therefore \quad BA' = V_1 \times t \qquad \text{--- (2)}$$

The secondary wavelets from A travel in the denser medium with a velocity V_2 and would cover a distance $(V_2 \times t)$ in t seconds. Therefore with A as centre and radius equal to $(V_2 \times t)$ draw an arc B'.

From A', draw a tangent plane touching the spherical arc tangentially at B'. Therefore A'B' is the secondary wavefront after t sec. This would advance in the direction of rays 1', 2' and 3', which are the corresponding refracted rays, perpendicular to A'B'.

Let r be the angle of refraction. As angle of refraction is equal to the angle which the refracted plane wavefront A'B' makes with the refracting surface AA', therefore $\angle AA'B' = \angle r$

In $\Delta AA'B$, $\quad \sin i = \dfrac{BA'}{AA'} = \dfrac{V_1 \times t}{AA'}$

In $\Delta AA'B'$, $\quad \sin r = \dfrac{AB'}{AA'} = \dfrac{V_2 \times t}{AA'}$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{V_1}{V_2} = {}^1\mu_2 \qquad \text{(by eq. (1))}$$

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \boxed{\mu_1 \sin i = \mu_2 \sin r}$$

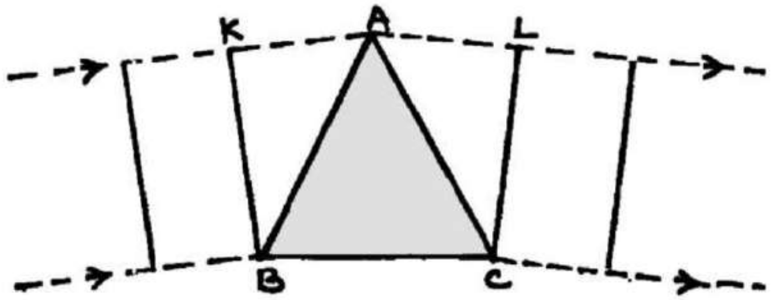
which Proves **Snell's Law** of refraction.

Behaviour of Prism towards Plane Wavefront-

Different parts of the wavefront travel different thickness of glass prism, maximum at the bottom and minimum at the top.

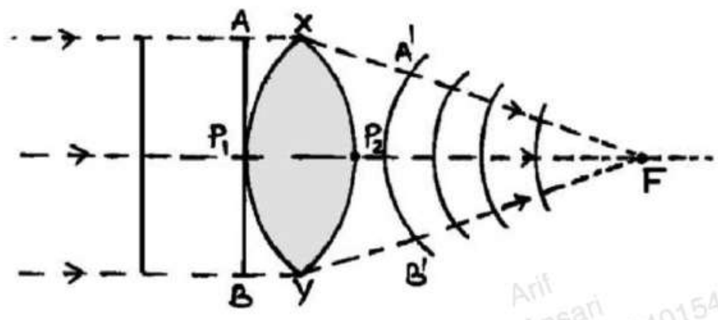
As speed of light in material of prism is less than the speed of light in air, therefore emerging plane wavefront is CL where-

$$\frac{KA + AL}{\text{speed of light in air}} = \frac{BC}{\text{speed of light in prism}}$$



It is clear from the figure that on passing through a prism, the rays bend towards the base of the prism.

Behaviour of a Lens towards Plane Wavefront

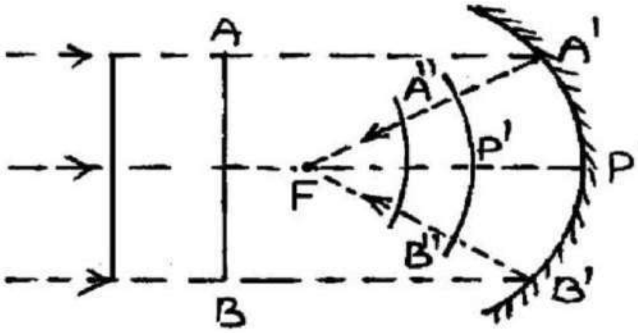


As light travels slower in lens material than in air, therefore refracted wavefront is converging spherical wavefront $A'P_2B'$. Hence on passing through a convex lens, the parallel rays tend to converge

Behaviour of a Spherical Mirror towards Plane Wave front

When a plane wavefront AB is incident on a concave mirror with pole at P. The centre of the wavefront has to travel the largest distance to P before it gets reflected, and the peripheral portion of wavefront has to travel the smallest distance before getting reflected.

Therefore the reflected wavefront is converging spherical wavefront A''P'B''.



Q. CBSE 2012 When monochromatic light is incident on a surface separating two media, why does the refracted light have the same frequency as that of the incident light?

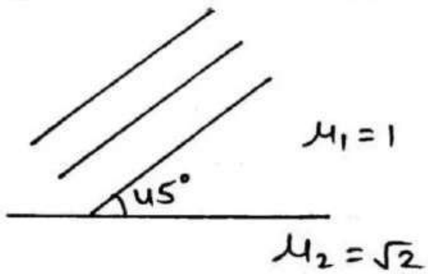
Sol: The frequency of an EM wave depends only on the source which produces it. The frequency is independent of the medium through which it travels, hence it does not change due to medium.

But the speed and wavelength depends on the medium through the wave travels.

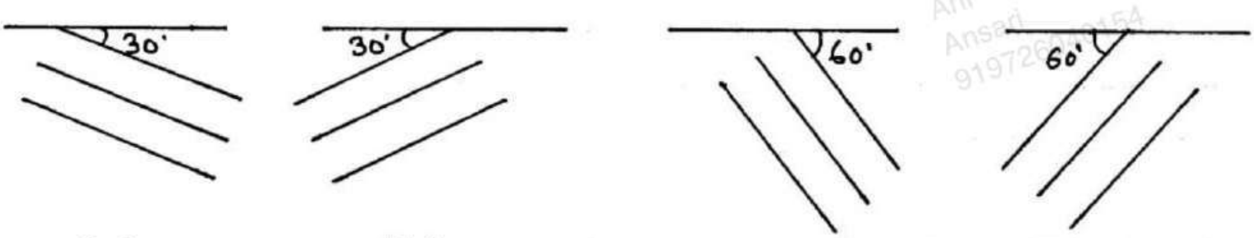
Q. CBSE 2011 When a light wave travels from a rarer to a denser medium, the speed decreases. Does it imply reduction in its energy? Explain.

Sol. No, the energy carried by the wave does not depend on its speed. Instead it depends on the amplitude of wave.

Q. In the following figure, a wavefront is falling on the boundary of two mediums. Find the refracted wavefront among the given options.



Options -



(a) (b) (c) (d)

Coherent Sources

The sources of light, which emit continuous light waves of the same wavelength, same frequency and in same phase or having a constant phase difference are called coherent sources.

*** Two independent sources of light cannot be coherent.**

This is because light is emitted from individual atoms, when they return to ground state after being excited by heat. Even the smallest source of light contains billions and billions of such atoms which obviously cannot emit light waves in the same phase.

Hence two independent sources of light whose light waves do not possess a constant phase difference are called non-coherent sources.

Two independent sources of light can not be coherent due to the following reasons -

(i) Even a tiniest source consists of millions of atoms and emission of light by them takes place independently.

(ii) The millions of atoms of a source cannot emit waves in the same phase.

(iii) Light is emitted by individual atoms and not by the bulk of matter acting as a whole.

Coherent sources of light can be obtained by deriving two sources from a source by -

(i) **Division of wavefront**

(ii) **Division of amplitude**

Superposition Principle-

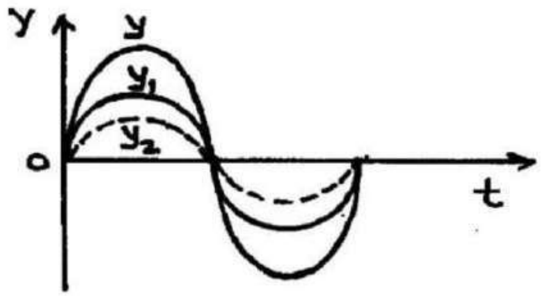
According to superposition principle, when two or more waves travelling through a medium superimpose one other, a new wave is formed in which resultant displacement ($\vec{y}$) at any instant is equal to the vector sum of the displacements due to individual waves ($\vec{y}_1, \vec{y}_2, \dots$) at that instant

Interference of Light-

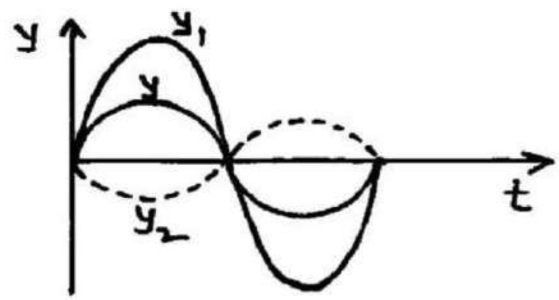
Interference of light is the phenomena of redistribution of light energy in a medium on account of superposition of light waves from two coherent sources.

* At a point, where the resultant intensity of light is maximum, interference is said to be constructive.

* At the points where the resultant intensity of light in minimum, the interference is said to be destructive.



$$y = y_1 + y_2$$

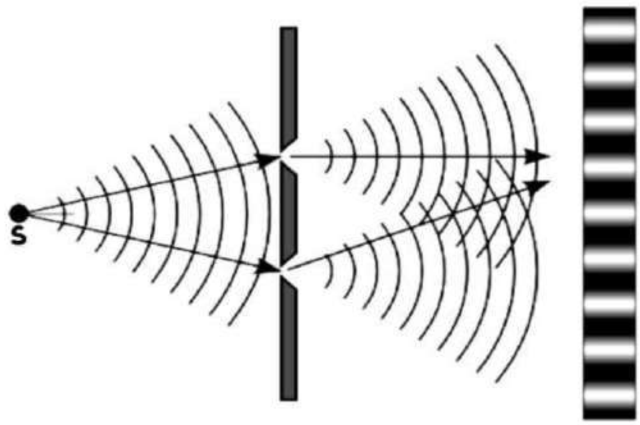


$$y = y_1 - y_2$$

Necessary Conditions for interference

(i) Both sources of light should be coherent (the waves emitted by them should have a constant phase difference and same frequency)

(ii) Both sources should be monochromatic (they should have equal frequency or wavelength)



(iii) The light waves emitted should preferably be of the same amplitude, so that the dark fringes are completely dark.

(iv) Two light sources must be very close to each other. If it is not so, then the path difference between the light waves reaching a particular point will be very large.

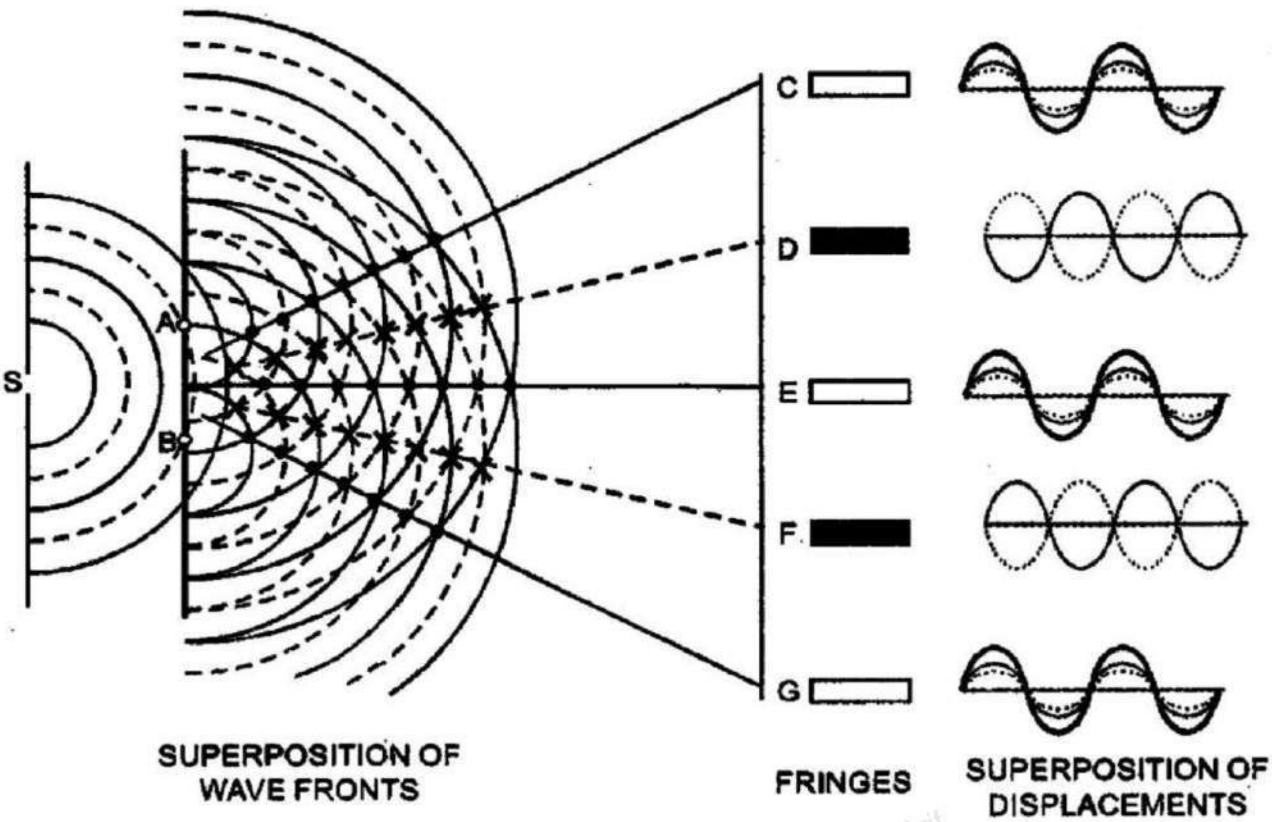
(v) The two light sources should be very narrow. A broad source of light is equivalent to a large number of point sources lying together.

(vi) Both waves should be in same state of polarisation if this is not then we got uniform illumination.

Young's double slit experiment

In this experiment S is a narrow slit (of width about 1 mm) illuminated by a monochromatic source of light. At a suitable distance (about 10cm) from S, there are two fine slits A and B about 0.5 mm apart.

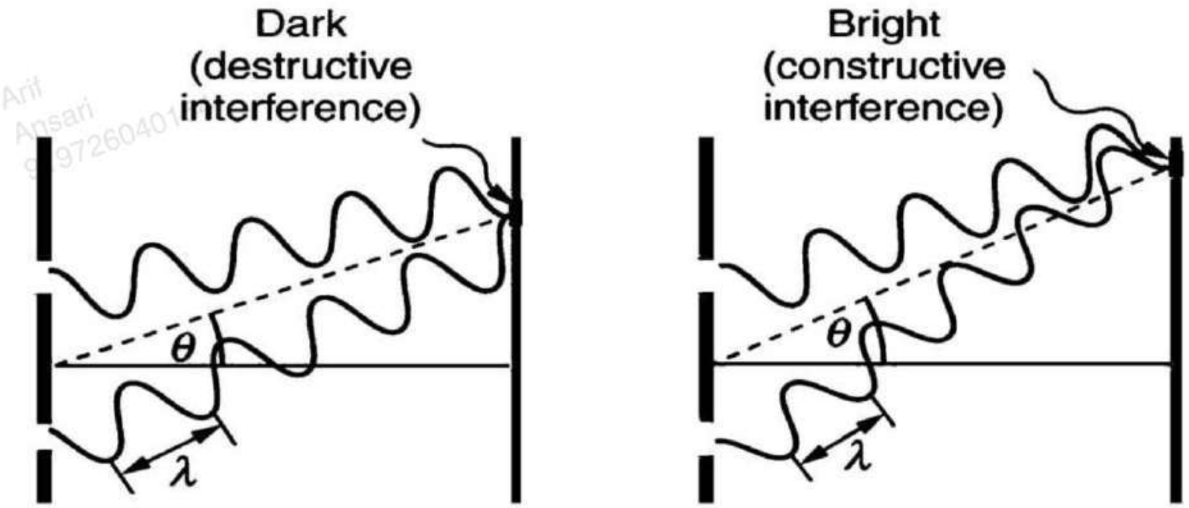
When a screen is placed at a large distance (about 2 m) from the slits A and B, alternate bright and dark fringes running parallel to the lengths of slits appear on the screen. These are the interference fringes.



Explanation -

According to Huygen's principle, the monochromatic source of light illuminating the slit S sends out spherical wavefronts.

Let the solid arcs represent the crests and the dotted arcs represents the troughs. These wavefront reach the slit A and B, which in turn becomes the sources of secondary wavelets. Thus the two waves of same amplitude and same frequency with zero phase difference are given out by A and B. These waves on superposition produce interference



The dots (•) represent the positions of constructive interference, where crest of one wave falls on crest of the other and trough falls on trough. The resultant amplitude and hence intensity of light is maximum at these positions.

Similarly the crosses (x) represent the positions of destructive interference, where crest of one wave falls on trough of the other and vice-versa. The resultant amplitude and hence intensity of light is minimum at these positions.

Thus we have bright fringes at C, E and G and dark fringes at D and F. These bright and dark fringes are placed alternately and they are equally spaced.

Q CBSE 2010

Show that the superposition of the waves originating from the two coherent sources S_1 and S_2 having displacement $Y_1 = a\cos\omega t$ and $Y_2 = a\cos(\omega t + \phi)$ at a point produce a resultant intensity -

$$I = 4a^2\cos^2 \phi/2$$

Hence, write the conditions for the appearance of dark and bright fringes.

Sol. $y_1 = a\cos\omega t \qquad y_2 = a\cos(\omega t + \phi)$

By Principle of superposition -

$$y = y_1 + y_2 = a\cos\omega t + a\cos(\omega t + \phi)$$

$$y = a\cos\omega t + a\cos\omega t\cos\phi - a\sin\omega t\sin\phi$$

$$y = a(1+\cos\phi)\cdot\cos\omega t + (-a\sin\phi)\sin\omega t$$

Let $a(1+\cos\phi) = A\cos\theta \quad \text{—— (1)}$

and $a\sin\phi = A\sin\theta \quad \text{—— (2)}$

$$\therefore \quad y = A\cos\theta\cos\omega t - A\sin\theta\sin\omega t$$

$$y = A\cos(\omega t + \theta)$$

Squaring and adding eq. (1) and (2), we get-

$$(A\cos\theta)^2 + (A\sin\theta)^2 = a^2(1+\cos\phi)^2 + (a\sin\phi)^2$$

$$A^2(\sin^2\theta + \cos^2\theta) = a^2[1 + \cos^2\phi + 2\cos\phi + \sin^2\phi]$$

$$A^2 = a^2 + a^2 + 2a^2\cos\phi$$

$$A^2 = 2a^2(1+\cos\phi)$$

$$A^2 = 2a\left(2\cos^2\frac{\phi}{2}\right) = 4a^2\cos^2\frac{\phi}{2}$$

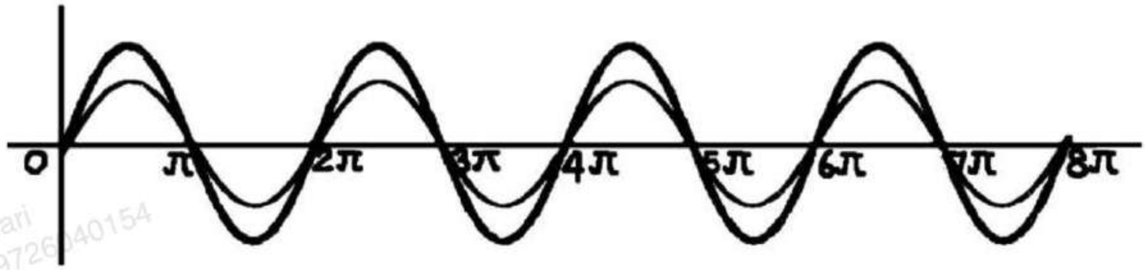
If I is the resultant intensity, then-

As $I = A^2$ $\qquad \boxed{I = 4a^2\cos^2\frac{\phi}{2}}$

Condition for bright fringe or Constructive Interference

When $\cos\left(\frac{\phi}{2}\right) = \pm 1$ then $I = I_0 = 4a^2$

Hence $\frac{\phi}{2} = n\pi$ or $\boxed{\phi = 2n\pi}$



Also $\boxed{\frac{\text{path difference}}{\lambda} = \frac{\text{Phase difference}}{2\pi}}$

or $x = \frac{\lambda}{2\pi} \times 2n\pi$

$\therefore$ $\boxed{\text{Path difference } x = n\lambda}$ $n = 0, 1, 2, \ldots$

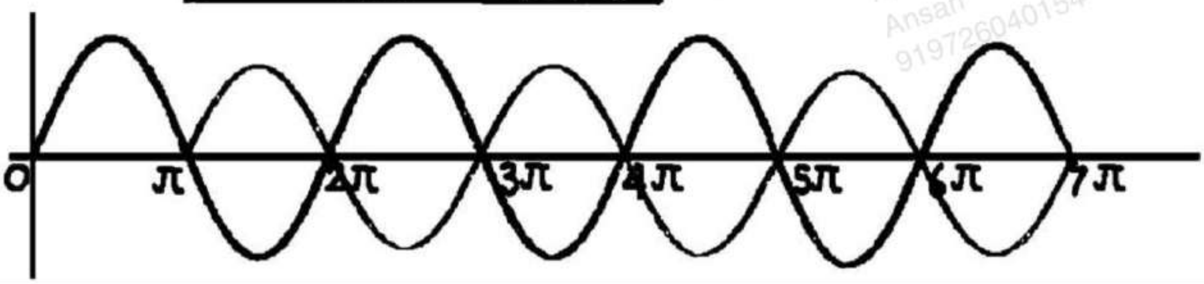
Hence the bright fringe is obtained when path difference of interfering waves in $n\lambda$ and phase difference is $2n\pi$.

Condition for dark fringe or destructive Interference

When $\cos\frac{\phi}{2} = 0$ then $I = 0$

Hence $\frac{\phi}{2} = (2n-1)\frac{\pi}{2}$

or $\boxed{\phi = (2n-1)\pi}$, where $n = 1, 2, \ldots$



Path difference $x = \frac{\lambda}{2\pi} \times \phi$

or $\boxed{\text{Path difference } x = (2n-1)\frac{\lambda}{2}}$

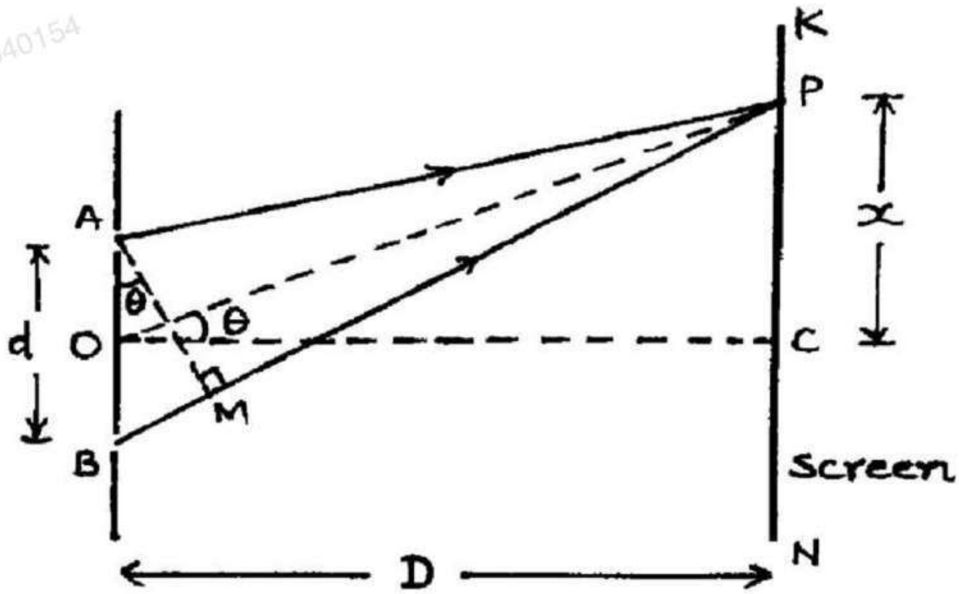
Hence dark fringes are obtained, when interfering wave have path difference is odd multiple of $\lambda/2$ and phase difference is odd multiple of π.

[redacted]

Expression for Fringe width in Interference

Suppose A and B are two fine slits and separated by a distance d. The slits are illuminated by a monochromatic light of wavelength λ. KN is a screen at a distance D from the slits.

The two waves starting from A and B superimpose on eachother, resulting in interference pattern on the screen.



Consider a point P on the screen, at a distance x from C.

The path difference between interfering waves is given by $= BP - AP$

$$\text{path difference} = BP - AP = BM$$
$$= d\sin\theta$$

and $\angle BAM = \angle POC$ (By geometry)

If θ is small, then $\sin\theta \approx \theta \approx \tan\theta$

$\therefore$ path difference $= BM = d\sin\theta \approx d\tan\theta$

$$= \frac{dx}{D} \qquad \left(\because \tan\theta = \frac{x}{D}\right)$$

For Constructive Interference -

path difference $= n\lambda$, where $n = 0, 1, 2, \ldots$

Hence $\dfrac{dx}{D} = n\lambda$ or $\boxed{x_n = \dfrac{n\lambda D}{d}}$

$\therefore$ Fringe width of dark fringe $\beta = x_{n+1} - x_n$

$$\beta = \frac{D\lambda}{d}(n+1) - \frac{D\lambda n}{d} = \frac{D\lambda}{d}$$

$$\boxed{\text{Fringe width of dark fringe } \beta = \frac{D\lambda}{d}}$$

For destructive Interference -

Path difference $= (2n-1)\dfrac{\lambda}{2}$, $(n = 1, 2, \ldots)$

Hence $(2n-1)\dfrac{\lambda}{2} = \dfrac{dx}{D}$

or $\boxed{x_n = \dfrac{(2n-1)D\lambda}{2d}}$

$\therefore$ Fringe width of bright Fringe $\beta = x_{n+1} - x_n$

$$\beta = \frac{(2(n+1)-1)D\lambda}{2d} - \frac{(2n-1)D\lambda}{2d} = \frac{D\lambda}{d}$$

$$\boxed{\text{Fringe width of bright fringe} = \frac{D\lambda}{d}}$$

★ Here we can see that, fringe width of dark fringe is equal to the fringe width of bright fringe.

* At the sites of constructive interference -

$$\boxed{I_{max} = (a + a)^2 = \text{Constant}}$$

Hence all bright interference bands have the same intensity.

* At the sites of destructive interference -

$$\boxed{I_{min} = (a - a)^2 = 0}$$

Hence all the dark fringes have same (zero) intensity.

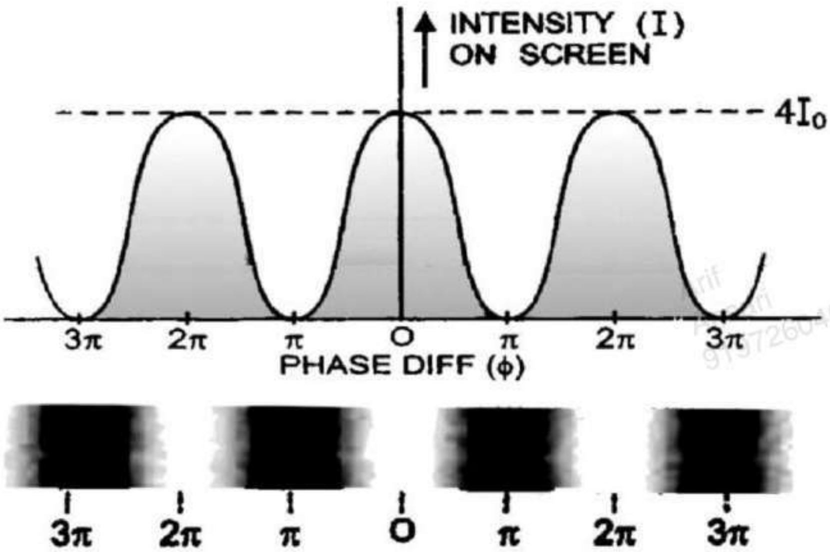
Special Case:

If $y_1 = a_1 \cos\omega t$

$y_2 = a_2 \cos(\omega t + \phi)$

$$\boxed{\begin{aligned} I &= I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi \\ a^2 &= a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi \end{aligned}}$$

$$\boxed{\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}} \qquad \boxed{\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2}$$



Some Important Points-

(1) The angular separation of interference fringes is $\frac{\lambda}{d}$. It is independent of position of the screen. Angular Position of n^{th} order -

$$\text{Bright Fringe} \quad \frac{x_n}{D} = \frac{n\lambda}{d} \qquad [n = 1, 2, \ldots.]$$

$$\text{Dark Fringe} \quad \frac{x_n}{D} = (2n-1)\frac{\lambda}{2d} \qquad [n = 1, 2, \ldots]$$

The angular separation of the fringes increases with increase in λ and decreases when d increases.

(2) In Young's double slit experiment, when the apparatus is dipped in liquid then wavelength λ will change, it will becomes $\frac{1}{\mu}$ times of its value in air.

$$\text{Fringe width in Medium} \quad \beta_m = \frac{D\lambda}{d\mu}$$

(3) When we use a source of white light, Containing light of different colours (i.e. different wavelengths λ), then at a particular point, Condition for constructive interference may be satisfied only for some particular value of λ.

Therefore, colour corresponding to this value of λ alone, shall be visible at that point.

Hence the interference fringes will be coloured. The centre of coloured fringe pattern will be white.

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As $\lambda_r > \lambda_v$ and $\beta = \frac{\lambda D}{d}$, therefore $\beta_r > \beta_v$ i.e. red fringe will be wider compared to the violet fringe.

Q. In Young's double slit experiment, the interfering waves have amplitudes in the ratio 3:2. Find the ratio of intensity of bright and dark fringes.

Answer: 25 : 1

Q. (Delhi 2016) In Young's double slit experiment, interference fringes are observed on a screen, kept at distance D from the slits. If the screen is moved towards the slits by 5×10^{-2} m, the change in fringe width is found to be 3×10^{-5} m. If the separation between the slits is 10^{-3} m. Calculate the wavelength of the light used.

Sol. Given that – $d = 10^{-3}$ m., $\Delta\beta = 3 \times 10^{-5}$ m

$\Delta D = 5 \times 10^{-2}$ m.

$\because$ fringe width $\beta = \frac{D\lambda}{d}$

$$\Rightarrow \Delta\beta = \frac{\lambda}{d} \times \Delta D$$

$$\Rightarrow \lambda = \frac{\Delta\beta \times d}{\Delta D} = \frac{10^{-3} \times 3 \times 10^{-5}}{5 \times 10^{-2}}$$

$$\lambda = 600 \times 10^{-9} \text{ m.} = 600 \text{ nm.}$$

Q. How would the angular separation of interference fringes in Young's double slit experiment change when the distance between the slits and screen is halved?

CBSE 2009

Sol. We Know that -

angular Separation $= \frac{\lambda}{d}$

Hence the angular separation of the interference fringes remains unchanged as it does not depends on D.

Q. How does the fringe width of interference fringes change, when the whole apparatus of Young's experiment is Kept in a liquid of refractive index 1·3?

CBSE 2008

Sol. We Know that -

fringe width in medium $\beta_m = \frac{D\lambda}{d\mu} = \frac{\beta}{\mu}$

Hence the new fringe width becomes $\frac{1}{1.3}$ times of original fringe width.

Q. Laser Light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 7·2mm Calculate the wavelength of another source of laser light which produce interference fringes separated by 8·1 mm using same pair of slits.

CBSE 2011

Sol. As Fringe width $(\beta) = \frac{D\lambda}{d}$

$\Rightarrow \quad \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$ [$\because$ D and d are same]

$\Rightarrow \quad \lambda_2 = \lambda_1 \times \frac{\beta_1}{\beta_2} = 630 \times 10^{-9} \times \frac{8.1 \times 10^{-3}}{7.2 \times 10^{-3}} = 708.75 \text{ nm}$

Q. The intensity at the central maxima (O) in a Young's double slit experiment is I_0. If the distance CP equals one third of fringe width of the pattern, show that the intensity at point P would be $I_0/4$.

CBSE 2011

Sol. Given that, the distance CP equals one-third of fringe width of the pattern. Hence

$$x = \frac{\beta}{3} = \frac{1}{3} \times \frac{D\lambda}{d} \quad \Rightarrow \quad \frac{xd}{D} = \frac{\lambda}{3}$$

$$\Rightarrow \text{Path difference} = BP - AP = \frac{dx}{D} = \frac{\lambda}{3}$$

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{Path difference}$$

$$\text{Hence Phase difference} = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

If intensity of the central fringe is I_0, then intensity at point P, where phase difference is ϕ is given by-

$$I = I_0 \cos^2 \phi$$

$$\Rightarrow \quad I = I_0 \cos^2\left(\frac{2\pi}{3}\right) = I_0 \left(-\frac{1}{2}\right)^2$$

or

$$\boxed{I = \frac{I_0}{4}}$$

Q. In Young's double slit experiment the two slits 0·15 mm apart are illuminated by monochromatic light of wavelength 450 nm. The screen is 1m awa from the slits. Find the distance of the second bright fringe and dark fringe from the central maximum.

CBSE 2010

Sol. Given that - $d = 0{\cdot}15\text{ mm} = 1{\cdot}5 \times 10^{-4}\text{ m}$.

$\lambda = 450\text{nm} = 450 \times 10^{-9}\text{m}$. $D = 1\text{ m}$.

Distance of n^{th} bright fringe from central fringe $= \frac{nD\lambda}{d}$

For second bright fringe $X_2 = \frac{2D\lambda}{d}$

$$X_2 = \frac{2 \times 1 \times 450 \times 10^{-9}}{1.5 \times 10^{-4}} = 6 \times 10^{-3} m.$$

The distance of n^{th} order dark fringe from central fringe is given by-

$$x'_n = (2n-1)\frac{D\lambda}{2d}$$

For 2^{nd} dark fringe $X'_2 = (2 \times 2 - 1)\frac{D\lambda}{2d}$

$$X'_2 = \frac{3 \times 1 \times 450 \times 10^{-9}}{2 \times 1.5 \times 10^{-4}} = 4.5 \times 10^{-3} m.$$

$\boxed{Q.}$ A beam of light, consisting of two wavelength 560 nm and 420 nm is used to obtain interference fringes in a Young's double slit experiment. Find the least distance from the central maximum, where the bright fringes, due to both the wavelengths coincide. The distance between the two slits is 4 mm and the screen is at a distance of 1 m from the slits.

Delhi 2020

Sol. Given that- $D = 1 m.$, $d = 4 \times 10^{-3} m.$

$\lambda_1 = 560 nm$, $\lambda_2 = 420 nm.$

Let nth order bright fringe of λ_1 coincide with $(n+1)$th order bright fringe of λ_2.

$$\Rightarrow (x_n)_{\lambda_1} = (x_{n+1})_{\lambda_2}$$

$$\Rightarrow \frac{nD\lambda_1}{d} = \frac{(n+1)D\lambda_2}{d}$$

$$\text{or} \quad n\lambda_1 = (n+1)\lambda_2$$

$$\Rightarrow \frac{n+1}{n} = \frac{\lambda_1}{\lambda_2} \quad \Rightarrow \quad 1 + \frac{1}{n} = \frac{560 \times 10^{-9}}{420 \times 10^{-9}}$$

$$\Rightarrow 1 + \frac{1}{n} = \frac{4}{3}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{3} \quad \Rightarrow n = 3$$

$\therefore$ Least distance from the central fringe where bright fringe of two wavelength coincides is

$$(x_3)_{\lambda_1} = \frac{3D\lambda_1}{d} = \frac{3 \times 1 \times 560 \times 10^{-9}}{4 \times 10^{-3}}$$

$$(x_3)_{\lambda_1} = 0.42 \times 10^{-3}\, m = 0.42\, mm.$$

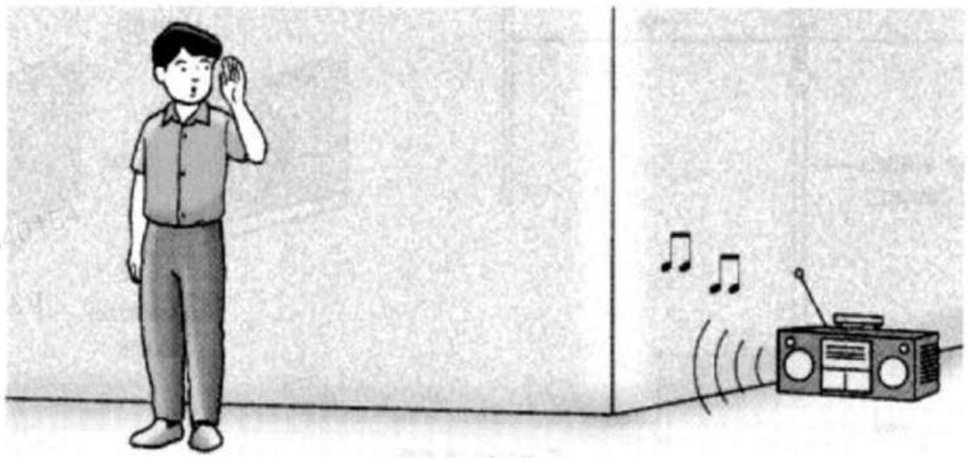
Hence 3rd order bright fringe of λ_1 and 4th order bright fringe of λ_2 coincide at 0.42 mm distance from the central fringe.

"You're much stronger than you think you are. Trust me"

- Spider-Man

Diffraction of sound waves-

Diffraction is the basic property of all types of waves. But it can only be observed when the size of obstacle or slit is of the order of the wavelength used.



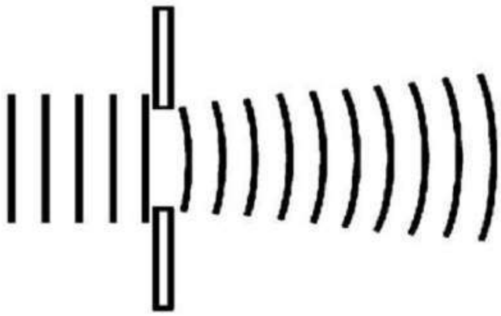
The wavelength of sound waves in air is of the order of 10 cm to 100 cm. As the wavelength of sound waves is of the order of normal obstacles or slits, so that diffraction of sound waves can be observed easily.

But the wavelength of light (4000 Å to 7800 Å) is very less and the normal slits and obstacles are bigger then this, so that the diffraction of light is not a normal incident.

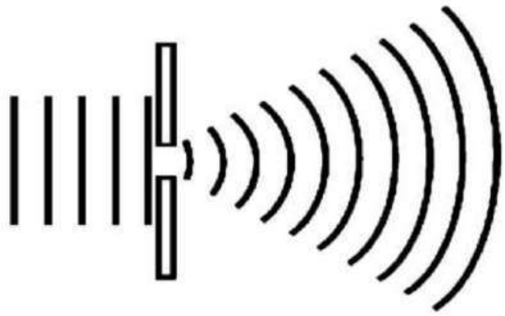
Diffraction of Light -

Diffraction of light is the phenomenon of bending of light around corners of an obstacle or aperture in the path of light. On account of this bending the light penetrates into the geometrical Shadow of the obstacle.

The diffraction becomes much more effective when the dimensions of the aperture or the obstacle are comparable to the wavelength of light.



Wide gap – small diffraction effect



Narrow gap – large diffraction effect

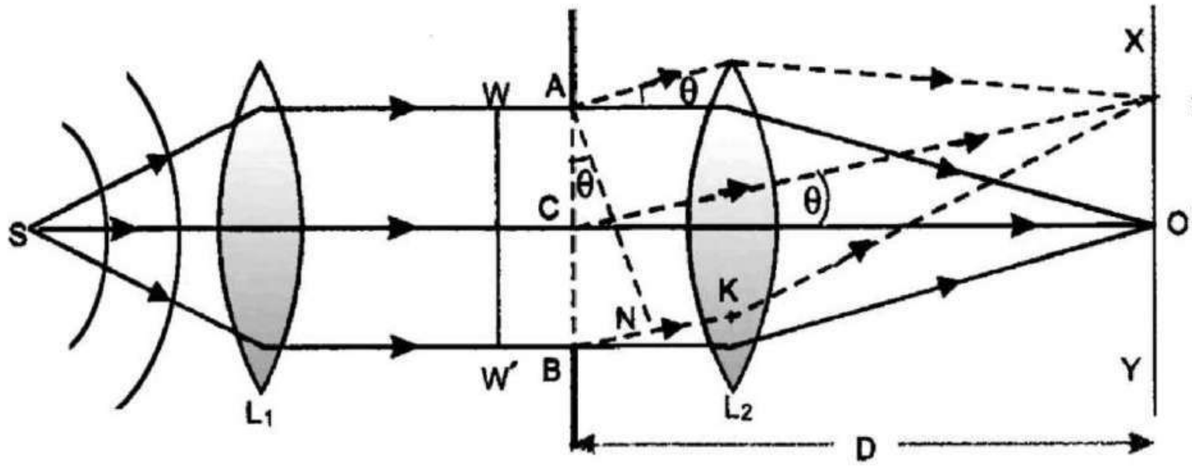
Out of the given two cases, diffraction is most pronounced when slit width (a) is smaller then wave-length (λ) i.e $a < \lambda$.

Diffraction of Light at a single slit-

In experimental set up S is a monochromatic light source held at the focus of a collimating lens L_1. A parallel beam of light emerging from the lens with a plane wavefront WW' is made to fall on a single slit AB of width a.

The diffraction pattern is focussed on the screen XY with the help of a convex lens L_2.

The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on both sides.



Explanation-

According to the Huygens principle each point on the plane wavefront AB sends out secondary wavelets in all the directions.

The secondary waves from points equidistant from the centre C of the slit lying in the portion CA and CB of wavefront travel the same distance in reaching O and hence the path difference between them is zero. These secondary waves superimpose each other, resulting in the maximum intensity at point O.

Consider a point P on the screen. Intensity at P will depend on the path difference between the secondary waves emitted from the corresponding point of the wavefront.

Now draw AN perpendicular to BK. Path difference between the secondary waves reaching P from A and B = BN

$$\text{Path difference } BN = AB \sin\theta = a \sin\theta$$

Position of Secondary Minima -

If this path difference is λ, then P will be point of minimum intensity. This is because whole wavefront can be considered to be devided into two equal halves CA and CB and if the path differen between the secondary waves from A and B is λ, then the path difference between the secondary wave from A and C reaching P will be $\lambda/2$.

Also for every point in the upper half AC, there is a corresponding point in the lower half CB for which the path difference between the secondary waves reaching P is $\lambda/2$. Thus destructive interference takes place at P and therefore, P is a point of first secondary minima.

Similarly, if path difference $BN = 2\lambda$, then the point P will be the position of second secondary minimum and so on.

Hence For n^{th} secondary minimum

$$\boxed{\text{Path difference} = a \sin\theta = n\lambda}$$

Position of Secondary Maxima -

If any other point P on the screen is such that the path difference $= a \sin\theta = 3\lambda/2$, then P will be the position of first secondary maximum.

Here we can assume that the wavefront be devided into three equal parts so that the path difference between secondary waves from corresponding points in the first two parts will be $\lambda/2$. They will give rise to destructive interference.

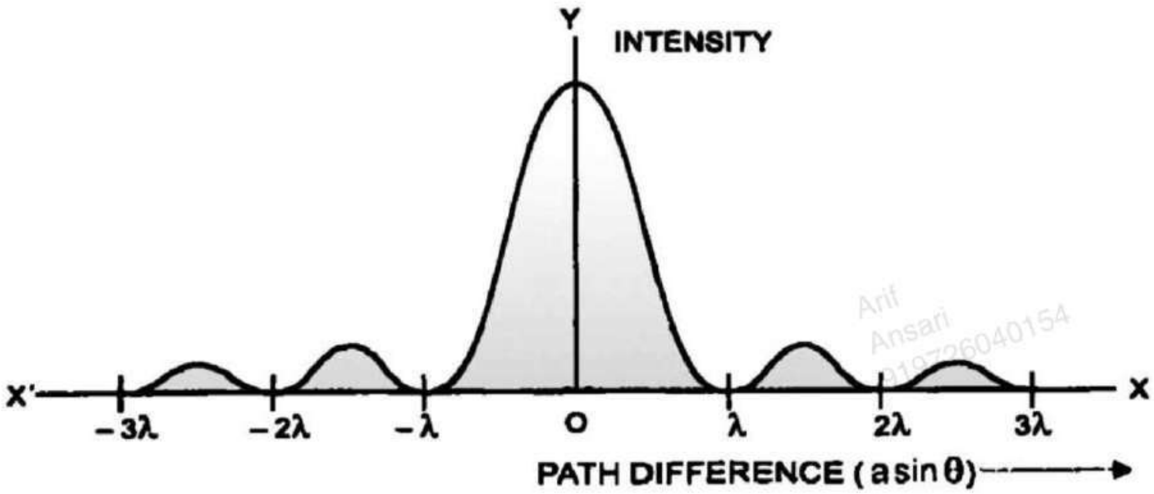
The secondary waves from third part, remain unused and therefore they superimpose each other and produce first secondary maxima.

Similarly if $BN = 5\lambda/2$, then we get second secondary maximum of lower intensity and so on.

Hence for n^{th} secondary maximum -

$$\boxed{a \sin\theta = (2n+1)\frac{\lambda}{2}}$$

Intensity distribution curve and diffraction Pattern



Width of Central Maximum -

The width of central maximum is the distance between first secondary minimum on either side of O.

If P is the position of 1st secondary minimum and $OP = x$ then -

$$a \sin\theta = 1 \times \lambda$$

or $$\sin\theta = \frac{\lambda}{a}$$

If θ is small, then $\sin\theta \simeq \tan\theta \simeq \theta$

Hence $$\tan\theta = \frac{\lambda}{a} \Rightarrow \frac{x}{D} = \frac{\lambda}{a}$$

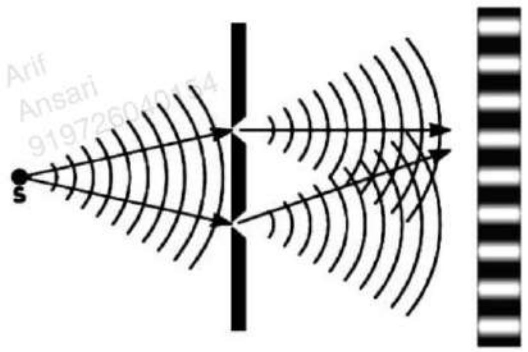
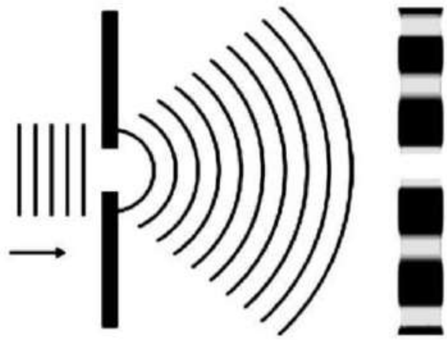
or $$x = \frac{D\lambda}{a}$$

$$\boxed{\text{Width of Central Maximum} = 2x = \frac{2D\lambda}{a}}$$

$$\boxed{\text{Angular width of Central Maximum} = 2\theta = \frac{2x}{D} = \frac{2\lambda}{a}}$$

Note- When source is emiting white light, then the diffraction pattern is coloured.
The central maximum is white, but other bands are coloured. As band width $\propto \lambda$, therefore, red band with higher wavelength is wider than the violet band with smaller wavelength.

Difference between Interference & Diffraction

	Interference	Diffraction
		
1.	It is due to super-position of two distinct waves coming from two coherent sources.	It is produced as a result of superposition of the secondary wavelets coming from different parts of the same wave front.
2.	In interference pattern all the bright fringes are of same intensity.	In diffraction pattern, all the bright bands are not of the same intensity.
3.	The width of the interference fringes may or may not be equal.	The fringe width of diffraction bands is unequal.

Q. CBSE 2018 How does the angular separation between fringes in single-slit diffraction experiment change when the distance of separation between the slit and screen is doubled?

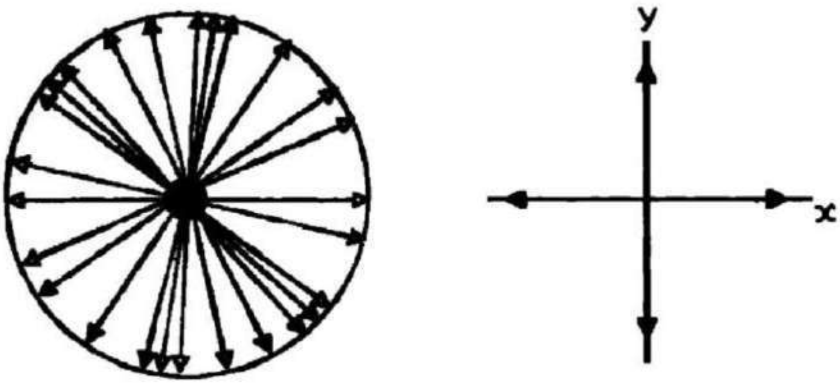
Sol. $\because$ Angular width $(\theta) = \frac{\lambda}{a}$

Here θ is independent of separation between slit and screen (D). Hence the angular separation remains the same.

Polarization of Light

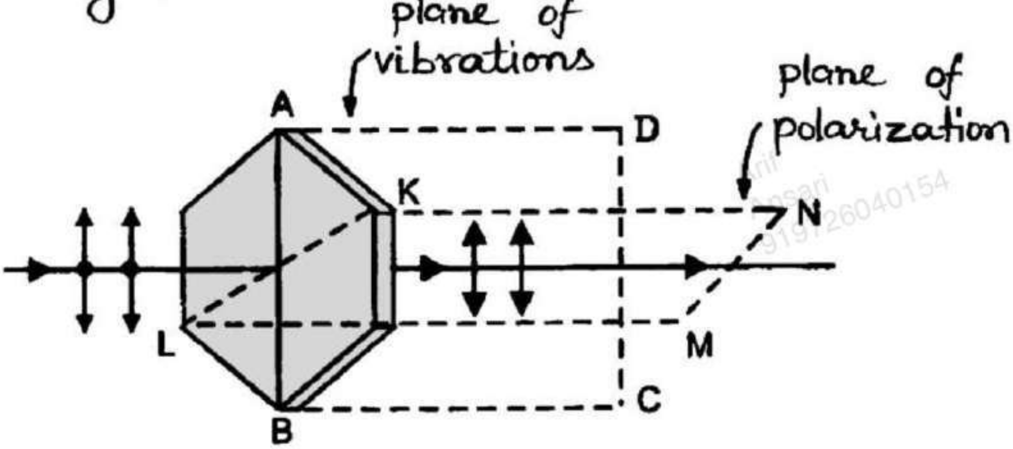
Light is an electromagnetic wave in which electric and magnetic field vectors vary sinusoidally, perpendicular to each other as well as perpendicular to the direction of propagation of Light wave.

The light propagated from common light source (sun or bulb) in a given direction consist of many independent waves whose plane of vibration are randomly oriented about the direction of propagation such light waves are said to be unpolarized.



When unpolarized light is passed through a tourmaline crystal cut with its face parallel to its crystallographic axis AB, then only those vibrations of light pass through the crystal which are parallel to AB. All other vibrations are absorbed.

The emergent light from the crystal which contain vibrations only in one direction is called plane polarized light.



Polarization -

The phenomena of restricting the vibrations of light (electric vector) in a particular direction perpendicular to the direction of wave motion is called polarization of light.

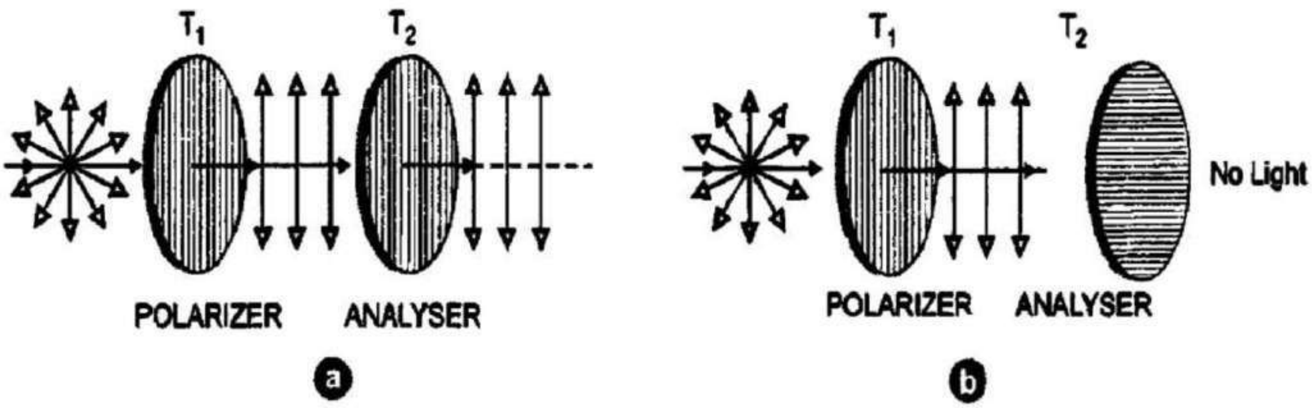
The tourmaline crystal acts as a polarizer.

The plane ABCD in which the vibrations of polarized light are confined is called the plane of vibration.

The plane KLMN which is perpendicular to the plane of vibration is defined as the plane of polarization.

Experimental Demonstration of Polarization-

In figure T_1 and T_2 are two thin plates of tourmaline.



A fine beam of ordinary light from the source s is passed through the plate T_1 and transmitted light is observed by the naked eye.

When the plate T_1 is rotated about the direction of propagation of light, the intensity of transmitted light remain the same.

Let the second plate T_2 be placed in the path of light transmitted from T_1. We observe that intensity of transmitted light by T_1 and T_2 remain unaffected only when T_1 and T_2 are set with their axes parallel.

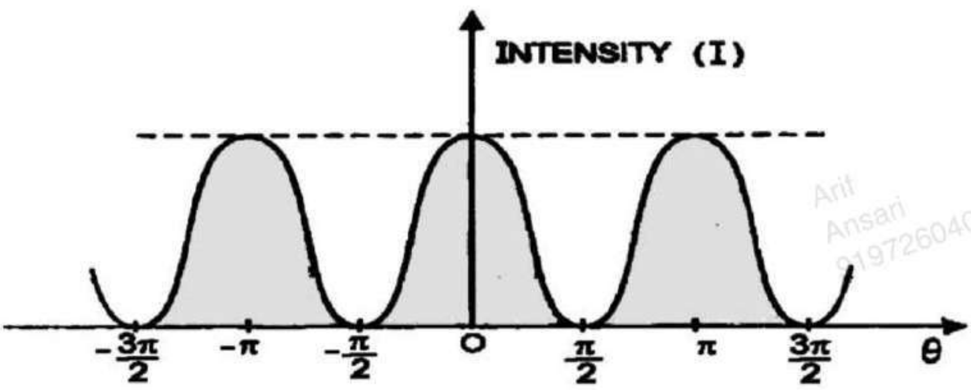
When T_2 is rotated gradually, intensity of light transmitted from T_2 goes on decreasing. As soon as the axes of the two crystals are at 90° to each other light is completely cut off. On rotating T_2 further, light reappears.

Hence light coming out of the crystal T_1 is said to be polarized and it has vibrations of electric vector which are restricted only in one direction (Parallel to the optic axis of crystal T_1).

Note -

The intensity of polarized light on passing through a tourmaline crystal changes, with the relative orientation of its crystallographic axes with that of polarizer, therefore light must consist of transverse waves.

Intensity of polarized light



θ = angle between plane of analyser and polarizer

Law of Malus -

According to Law of Malus, when a beam of completely plane polarized light is incident on an analyzer, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of the cosine of the angle (θ) between plane of transmission of analyser and polarizer.

$$\boxed{I \propto \cos^2\theta}$$

Proof -

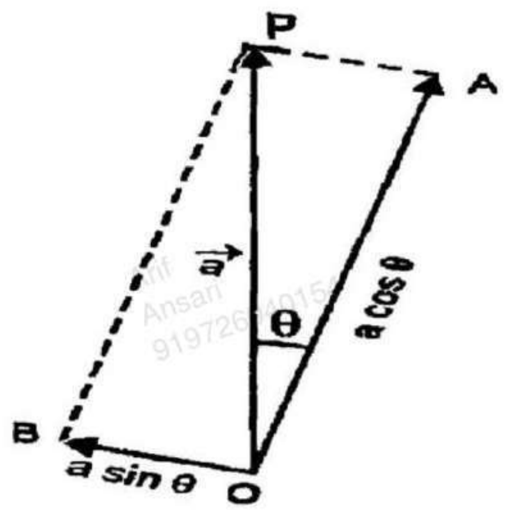
Suppose $\vec{a}$ is amplitude of electric vector transmitted by the polarizer along OP. The plane of analyser OA makes an angle θ with OP. The two rectangular components of $\vec{a}$ are $a\cos\theta$ and $a\sin\theta$, which are parallel and perpendicular to plane of transmission of analyser respectively.

As the component transmitted through analyser is only $a\cos\theta$, therefore, intensity of light transmitted from analyser is -

$$I \propto a^2\cos^2\theta$$

Hence $\boxed{I = I_0\cos^2\theta}$ or $\boxed{I \propto \cos^2\theta}$

I_0 = Intensity of incident light



(i) When polariser and analyser are parallel ($\theta = 0°$ or $180°$)

$$\boxed{I = I_0}$$

(ii) When polariser and analyser are perpendicular ($\theta = 90°$)

$$\boxed{I = 0}$$

Q. Between two polaroids placed in crossed position, a third polaroid is introduced. The axis of the third polaroid makes an angle of 30° with the axis of the first polaroid. Find intensity of transmitted light from the system assuming I_0 to be the intensity of the polarized light incident at the first polaroid.

CBSE 2011

Sol. The intensity of transmitted light from P_1 is -

$$I_1 = \frac{I_0}{2}$$

The intensity of transmitted light from P_2 is -

$$I_2 = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2 30^\circ$$

$$= \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow I_2 = \frac{3I_0}{8}$$

The intensity of transmitted light from P_3 is -

$$I_3 = I_2[\cos(90^\circ - \theta)]^2$$

$$= \frac{3I_0}{8}[\cos(90^\circ - 30^\circ)]^2$$

$$I_3 = \frac{3I_0}{8} \times \left(\frac{1}{2}\right)^2 \Rightarrow \boxed{I_3 = \frac{3I_0}{32}}$$

Q. (CBSE 2010) Light from a sodium Lamp is passed through two polaroid sheets P_1 and P_2 Kept one after the other. Keeping P_1 fixed P_2 is rotated so that its pass axis can be at different angles θ, with respect to the pass axis P_1. The given data for the intensity of light coming out of P_2 as a function of the angle θ.

S.N.		1	2	3	4	5
1.	θ	$0°$	$30°$	$45°$	$60°$	$90°$
2.	I	$\frac{I_0}{2}$	$\frac{3I_0}{8}$	$\frac{I_0}{2\sqrt{2}}$	$\frac{I_0}{8}$	0

One of these observations is not in agreement with the expected theoretical variation of I. Identify this observation and write the correct expression.

Sol. The observation 3 is not in agreement with the expected theoretical variation of I.

For angle $\theta = 45°$, intensity of transmitted light from P_2 is–

$$I = \frac{I_0}{2}\cos^2 45°$$

$$I = \frac{I_0}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 \Rightarrow \boxed{I = \frac{I_0}{4}}$$

Q. (CBSE 2019) Two polaroids A and B are Kept in crossed position How should a third polaroid C be kept between the So that the intensity of polarized light transmitted by polaroid B reduces to $\frac{1}{8}$th of the intensity of unpolarized light incident on A.?

Sol. Let θ be the angle between the pass axis of A and C.

Intensity of light passing through $A = \frac{I_0}{2}$

Intensity of light passing through C is -

$$I = \frac{I_0}{2} \cos^2\theta$$

Intensity of light passing through B is -

$$I' = \left(\frac{I_0}{2} \cos^2\theta\right) \cos^2(90^\circ - \theta)$$

$$I' = \frac{I_0}{2} (\cos\theta \sin\theta)^2$$

But given that $\frac{I_0}{2} (\cos\theta \sin\theta)^2 = \frac{I_0}{8}$

$$(\sin\theta \cos\theta)^2 = \frac{1}{4}$$

$$\left(\frac{\sin 2\theta}{2}\right)^2 = \frac{1}{4}$$

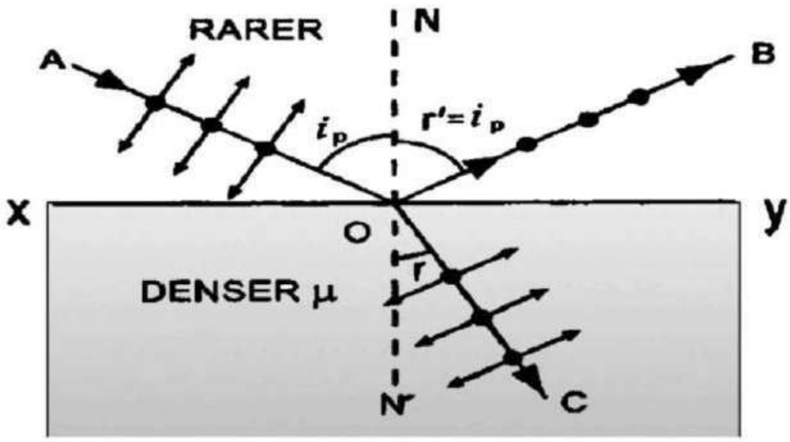
$$\sin 2\theta = 1 \quad \Rightarrow \quad 2\theta = \sin^{-1}(1)$$

$$\theta = 45^\circ$$

Polarization by Reflection-

When unpolarized light is reflected from a surface, the reflected light may be completely polarized, partially polarized or unpolarized. This would depend on the angle of incidence.

If angle of incidence is $0°$ or $90°$, then the reflected beam remain unpolarized. For angles of incidence between $0°$ and $90°$, the reflected beam is polarized to varying degree.



The angle of incidence at which the reflected light is completely plane polarized is called polarizing angle or Brewster's angle. It is represented by i_p.

The value of i_p depends on the wavelength of light used.

As shown in figure when unpolarized light is incident along AO at $\angle i_p$ on the interface XY separating air from a medium of refractive index μ. The reflected light along OB is completely plane polarized.

The vibrations of electric vector perpendicular to the plane of incidence remain always parallel to the reflecting surface, whatever be the angle of incidence. Therefore condition of their reflection is not changed with the change in the angle of incidence

However the other set of oscillations of electric vector in the plane of incidence make different angles with the reflecting surface, as angle of incidence of the unpolarized light is changed. At polarizing angle (i_p), most of these vibrations of electric vector get transmitted and are not reflected.

Hence refracted light along OC is a mixture of polarized light and unpolarized light i.e. it is partially polarized.

Brewster's Law -

According to this Law, when unpolarized light is incident at polarizing angle (i_p) on an interface separating air from a medium of refractive index μ, then the reflected light is fully polarized (Perpendicular to the plane of incidence), provided

$$\boxed{\mu = \tan i_p}$$

This relation represents Brewster's Law.

★ when light is incident at polarizing angle, the reflected component along OB and refracted component along OC are mutually perpendicular to eachother.

Proof of Brewster's Law -

In figure $\quad \angle BOY - \angle YOC = 90^\circ$

$$(90^\circ - i_p) + (90^\circ - r) = 90^\circ$$

or $\qquad 90^\circ - i_p = r$ (r = angle of refraction)

According to snell's Law -

$$\mu = \frac{\sin i}{\sin r}$$

when $i = i_p$ and $r = (90^\circ - i_p)$

$$\therefore \quad \mu = \frac{\sin i_p}{\sin(90^\circ - i_p)}$$

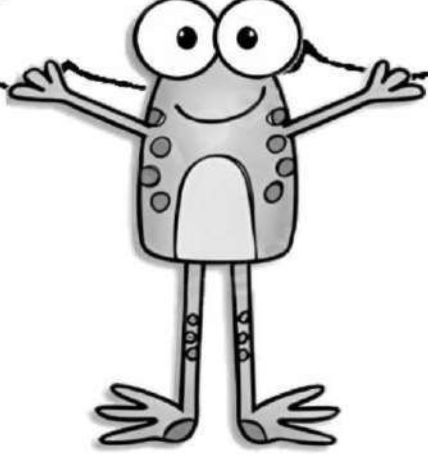
or $\qquad \mu = \frac{\sin i_p}{\cos i_p} = \tan i_p$

$\Rightarrow \boxed{\mu = \tan i_p}$ This proves Brewster's Law.

For air-water interface ${}^a\mu_w = 4/3$

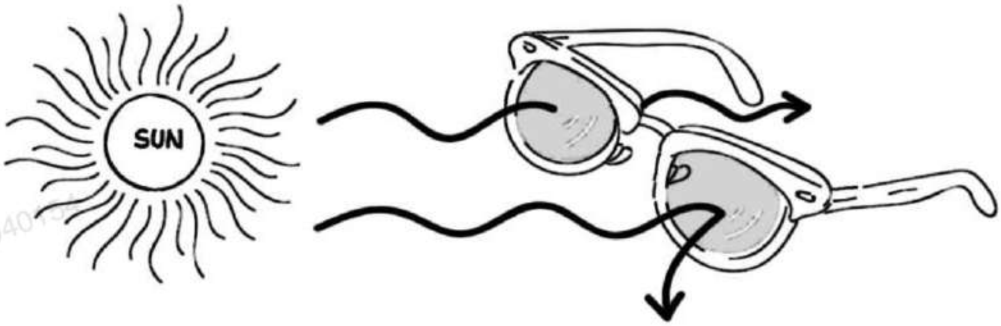
$\therefore \quad i_p = \tan^{-1}\left(\frac{4}{3}\right) \Rightarrow i_p = 53.1^\circ$

Thus light reflected in air from surface of water will be completely polarised, when angle of incidence of unpolarized light on the surface of water is 53.1°



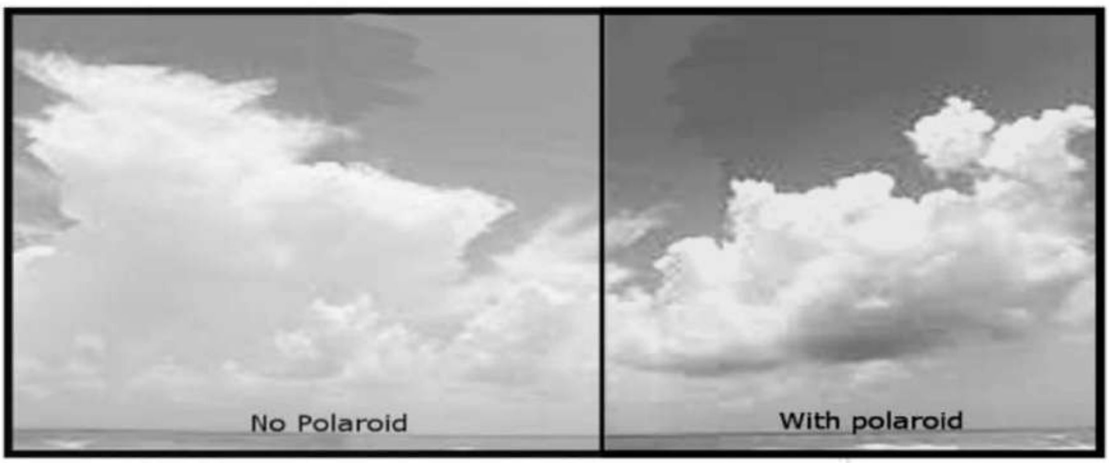
Use of plane polarized Light and Polaroids -

(1.) The major use of polaroids is to avoid glare of light. When we use polarized sun glasses with their vibration plane - verticle, the most of the polarized light reflected from glazed surfaces is cut off.



(2.) To avoid the dazzling light of a car approaching from the opposite side during night driving, polaroids are fitted on the wind shield and on the cover glasses of head lights of each car.

(3.) Clear photographs of white clouds are obtained by fitting polaroids in front of the camera lens.



(4.) Polaroids are useful in three dimensional motion pictures

(5.) In CD players, polarized laser beam acts as needle for producing sound from compact disc (CD).

Q.1. Yellow light with wavelength 0.5 μm in air suffers refraction in a medium in which velocity of light is $2\times10^8 m/s$. Then the wavelength of the light in the medium would be.

Solution :

$$\frac{\lambda_1}{\lambda_2}=\frac{v_1}{v_2}$$

Here, $\lambda_1=0.5\,\mu m, v_1=3\times10^8 m/s$

$\lambda_2=?(x), v_2=2\times10^8 m/s$

$$\frac{0.5}{x}=\frac{3\times10^8}{2\times10^8}$$

$$\Rightarrow x=0.33\,\mu m$$

Note : The frequency remains unchanged.

Q.2. Light waves from two coherent sources having intensity ratio 81 : 1 produce interference. Then, the ratio of maxima and minima in the interference pattern will be

Solution :

Given, $\frac{I_1}{I_2}=\frac{A_1^2}{A_2^2}=\frac{81}{1}$

$\therefore \frac{A_1}{A_2}=\frac{9}{1}$ **or** $A_1=9A_2$ .... (1)

$$\therefore \frac{I_{max}}{I_{min}}=\frac{(A_1+A_2)^2}{(A_1-A_2)^2}$$

From Eq. (i), we get

$$\frac{I_{max}}{I_{min}}=\frac{(9A_2+A_2)^2}{(9A_2-A_2)^2}=\frac{(10)^2}{(8)^2}=\frac{25}{16}$$

Q.3. Compare the intensities of two points located at respective distance $\frac{\beta}{4}$ and $\frac{\beta}{3}$ from the central maxima in a interference of YDSE (β is the fringe width)

Solution :

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\left(\frac{d}{D}\frac{\beta}{4}\right) = \frac{2\pi}{\lambda}\left(\frac{d}{D}\frac{\lambda D}{4d}\right)$$

$$\therefore\ \Delta\phi = \frac{2\pi}{4} = \frac{\pi}{2} \Rightarrow I = 4I_0\cos^2\left(\frac{\pi}{4}\right)$$

Similarly

$$\Delta\phi = \frac{2\pi}{3} \Rightarrow I = 4I_0\cos^2\left(\frac{2\pi}{2\times3}\right) = I_0$$

$$\therefore \text{ required ratio} = \boxed{2:1}$$

Q.4. A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit.

Solution :

$$\theta = \frac{y}{D}, \theta = \frac{2.5\times10^{-3}}{1}\textit{radian}$$

Now, $a\sin\theta = n\lambda$

Since θ is very small, therefore $\sin\theta = \theta$.

$$\text{or } a = \frac{n\lambda}{\theta} = \frac{1\times500\times10^{-9}}{2.5\times10^{-3}}m$$

$$= 2\times10^{-4}\,m = 0.2\,mm$$

Q.5. A screen is placed 50 cm from a single slit, which is illuminated with 6000 Å light, If distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit?

Solution :

In case of diffraction at single slit, the position of minima is given by $a\sin\theta = n\lambda$. Where a is the aperture size and for small θ:

$$\sin\theta = \theta = (y/D)$$

$$\therefore\ a\left(\frac{y}{D}\right) = n\lambda,\ i.e., y = \frac{D}{a}(n\lambda)$$

So that, $y_3 - y_1 = \frac{D}{a}(3\lambda - \lambda) = \frac{D}{a}(2\lambda)$ and hence,

$$a = \frac{0.50\times(2\times6\times10^{-7})}{3\times10^{-3}} = 2\times10^{-4}\,m$$

$$= 0.2\,mm$$

Q.6. In a single slit diffraction experiment first minimum for $\lambda_1 = 660\,nm$ coincides with first maxima for wavelength λ_2. Calculate λ_2.

Solution :

Position of minima in diffraction pattern is given by;

$$a\sin\theta = n\lambda$$

For first minima of λ_1, we have

$$a\sin\theta_1 = (1)\lambda_1 \quad \text{or} \quad \sin\theta_1 = \frac{\lambda_1}{a} \quad .....(i)$$

The first maxima approximately lies between first and second minima. For wavelength λ_2 its position will be

$$a\sin\theta_2 = \frac{3}{2}\lambda_2 \ \therefore\ \sin\theta_2 = \frac{3\lambda_2}{2a} \quad ...... (ii)$$

The two will coincide if,

$$\theta_1 = \theta_2 \quad \text{or} \quad \sin\theta_1 = \sin\theta_2$$

$$\therefore \quad \frac{\lambda_1}{a} = \frac{3\lambda_2}{2a} \quad \text{or}$$

$$\lambda_2 = \frac{2}{3}\lambda_1 = \frac{2}{3} \times 660\,nm = 440\,nm$$

Q.7. Two slits are made one millimeter apart and the screen is placed one meter away. What should the width of each slit be to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern.

Solution :

We have $a\theta = \lambda$ (or) $\theta = \frac{\lambda}{a}$

(a = width of each slit)

$$10\frac{\lambda}{d} = 2\frac{\lambda}{a}$$

$$\therefore \quad a = \frac{d}{5} = \frac{1}{5} = 0.2\,mm$$

Q.8. Assume that light of wavelength 6000 Å is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of 100 inch?

Solution :

A 100 inch telescope implies that

a = 100 inch = 254 cm. Thus if,

$\lambda \approx 6000\,\text{Å} = 6 \times 10^{-5}\,cm$ then,

$$\Delta\theta = \frac{1.22\lambda}{a} \approx 2.9 \times 10^{-7} \quad \text{radians}$$

Q.9. When light of a certain wavelength is incident on a plane surface of a material at a glancing angle 30⁰, the reflected light is found to be completely plane polarized determine.

a) refractive index of given material and

b) angle of refraction.

Solution :

a) Angle of incident light with the surface is 30^0. The angle of incidence $= 90^0 - 30^0 = 60^0$. Since reflected light is completely polarized, therefore incidence takes place at polarizing angle of incidence θ_p.

$$\therefore \theta_p = 60^0$$

Using Brewster's law

$$\mu = \tan\theta_p = \tan 60^0 \Rightarrow \mu = \sqrt{3}$$

b) From Snell's law

$$\mu = \frac{\sin i}{\sin r} \qquad \therefore \sqrt{3} = \frac{\sin 60^0}{\sin r}$$

or $\quad \sin r = \dfrac{\sqrt{3}}{2} \times \dfrac{1}{\sqrt{3}} = \dfrac{1}{2}, \quad r = 30^0$.

Q.10. Two polaroids are oriented with their transmission axes making an angle of 30° with each other. What fraction of incident un polarized light is transmitted?

Solution :

If unpolarized light is passed through a polariod P_1 its intensity will become half.

So $I_1 = \frac{1}{2} I_0$ with vibrations parallel to the axis of P_1. Now this light will pass through the second Polaroid P_2 (Analyser) whose axis is inclined at an angle of 30° to the axis of P_1 and hence vibrations of I_1. So in accordance with Malus law, the intensity of light emerging from P_2 will be

$$I_1 = I_2 \cos^2 30° = \left(\frac{1}{2} I_0\right)\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_0$$

So the fractional transmitted light $\frac{I_2}{I_0} = \frac{3}{8}$